Google AI Quantum

Quantum Simulations at Google

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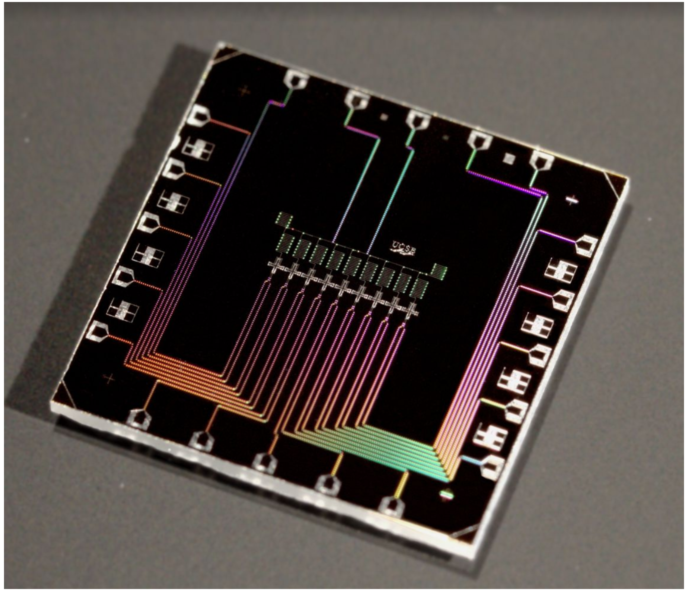
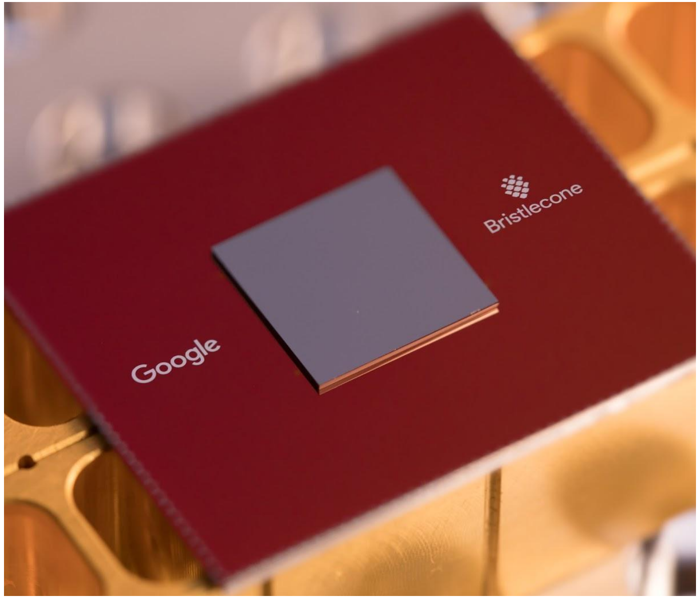
“Next Steps in Quantum Science for HEP” at Fermilab

September 12, 2018

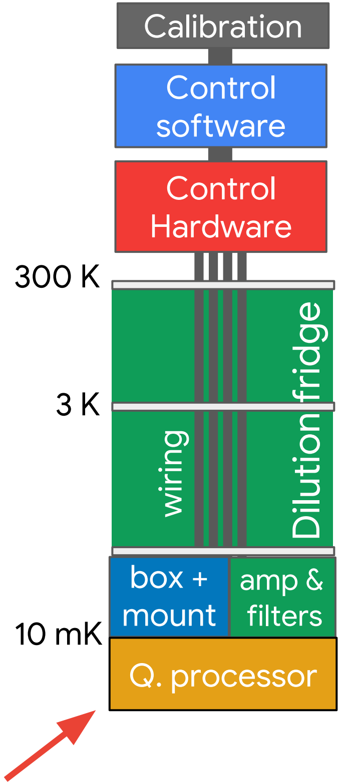
Quantum processors



Physics lab: 2015
9 qubits
1D n.n. coupled



Google: 2018
Bristlecone: 72 qubits
2D n.n. coupled



* Other processors architectures also in development!

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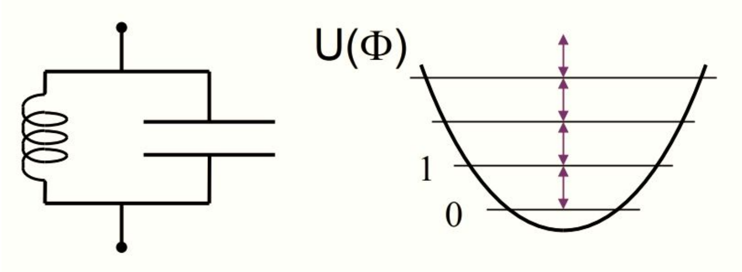
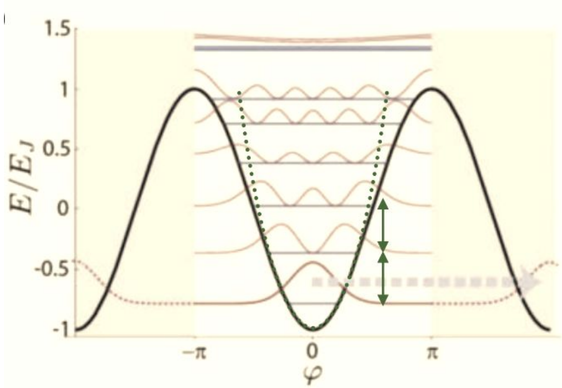
Superconducting qubits

Josephson junction: spectrum is unevenly spaced due to nonlinearity.





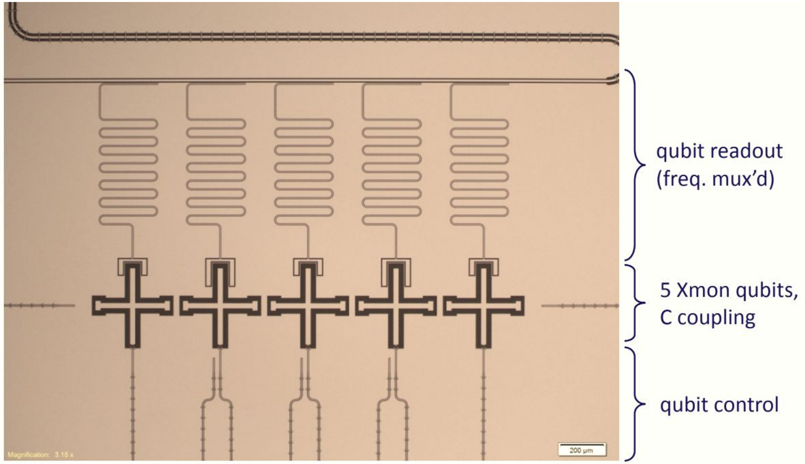
LC oscillator: evenly spaced spectrum makes it hard to exclude higher excited states from the qubit subspace.

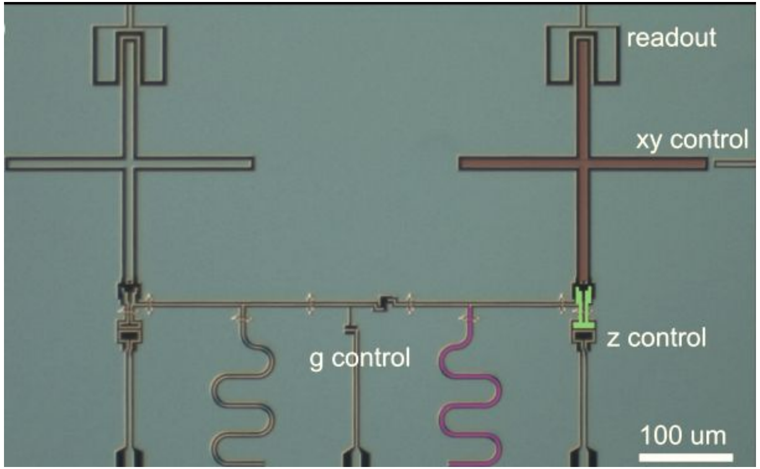
Transmon comes from **trans**mission line and junction plas**mon** mode. The original idea is to get around dephasing due to charge fluctuations by shunting the junction with a transmission line (later changed to a capacitor).

X-mon and G-mon

X-mons: capacitively coupled, coupling strengths cannot be changed or turn off, qubits are parked at different frequencies

G-mons: inductively coupled, coupling strengths can be controlled, can turn off the coupling completely

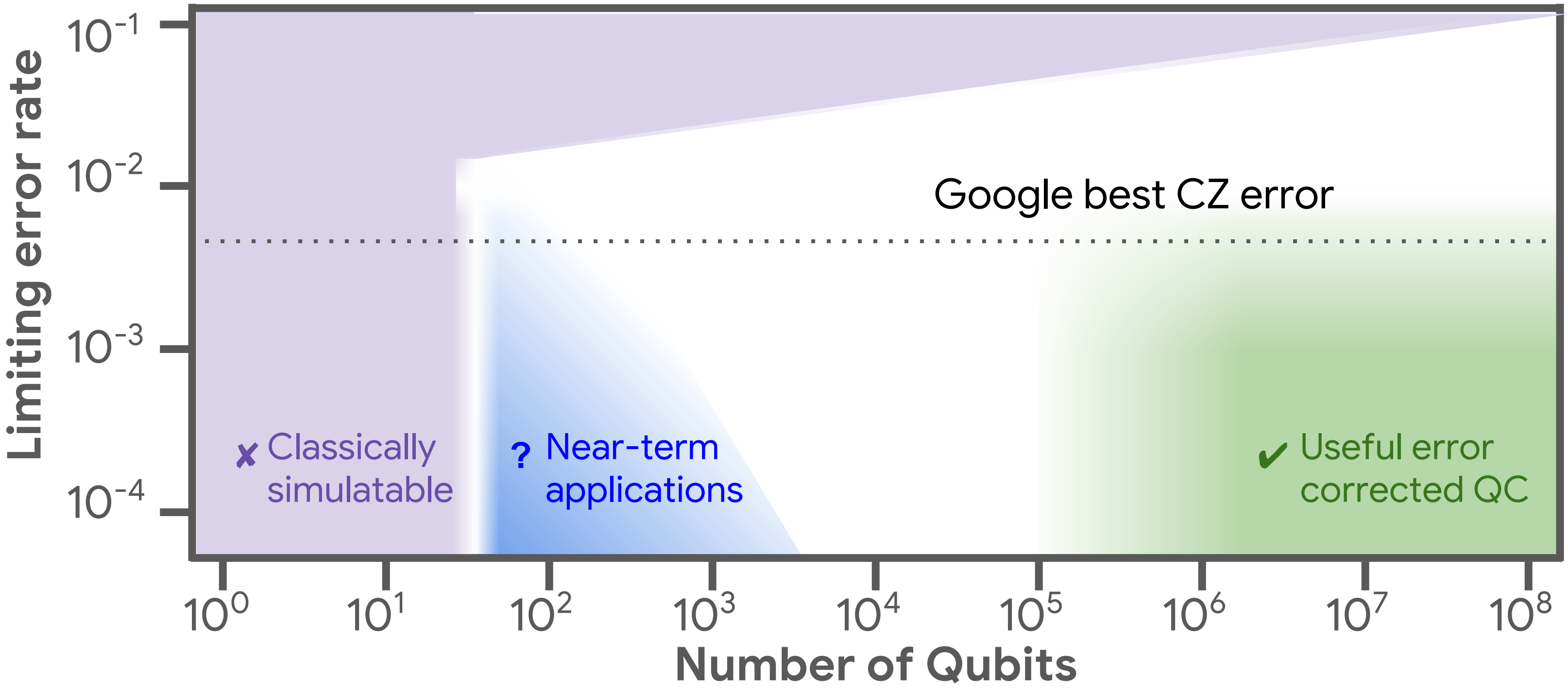




Some data for Google's qubits

- Operational Temperature ~ 15mK
- Qubit initialization takes 7μs and has fidelity > 0.99
- Qubit readout takes 1μs and has fidelity > 0.95
- Typical two-qubit gate time ~ 30ns
- T_1 ~ 20μs and T_2^* ~ 5μs.

Hardware: Errors and scaling



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Near-term Quantum goals

1. Demonstrate “Quantum Supremacy”

   Solve a problem, not necessarily useful, better than a supercomputer

2. Deliver processors to Cloud

   First quantum hardware product launch (2019)

3. Useful Algorithms & Applications

   Quantum simulation, optimization, quantum machine learning

Software pipeline

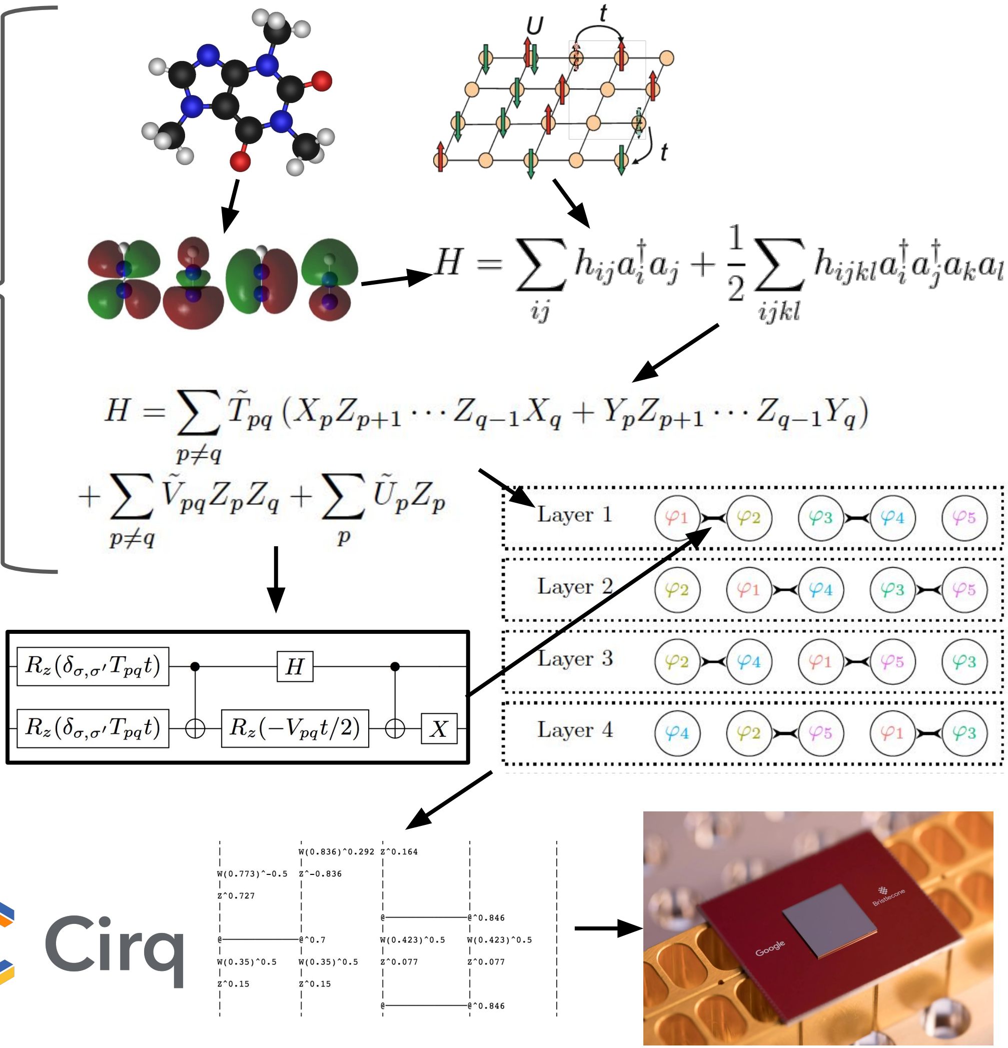
OpenFermion

- Compute basis functions
- Obtain Hamiltonians
- Exploit symmetries
- Map to qubits

OpenFermion-Cirq

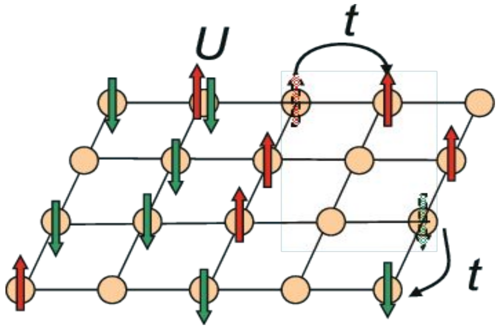
- Derive problem specific gates
- Layout algorithm primitives
- Encapsulate VQE ansatz
- Compile to hardware

Cirq



Fermi-Hubbard model

The Hubbard model is the simplest model to describe cuprate superconductor **qualitatively**.



hopping

on-site interaction

$$\mathcal{H}_{\mathrm{FH}} = - \sum_{\langle j,k \rangle,\sigma} t_{jk} \left(c^{\dagger}_{j,\sigma} c_{k,\sigma} + \mathrm{h.c.} \right) + U \sum_{j} n_{j,\uparrow} n_{j,\downarrow} + \sum_{j,\sigma} (\epsilon_j - \mu) n_{j,\sigma} - \sum_{j} h_j (n_{j,\uparrow} - n_{j,\downarrow}) \, ,$$

local field

chemical potential

magnetic field

Long-range Coulomb interaction is replaced by local **on-site interaction**.

where σ = ↑, ↓ denotes the two spin states and $n_{j,\sigma} = c^{\dagger}_{j,\sigma} c_{j,\sigma}$ denotes the occupation number operator of site j with spin σ.

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D-Wave pairing in momentum space

In the momentum space, the states k↑ and -k↓ are paired together.

$$\mathcal{H}_{\mathrm{DW}} = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}}\, c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} - \sum_{\mathbf{k}} \Delta_{\mathbf{k}} \big(c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \mathrm{h.c.} \big),$$

The superconducting gap is anisotropic

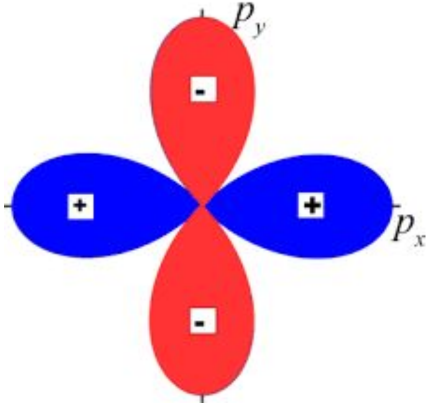
$$\xi_{\mathbf{k}} = -2t \Big(\cos(2\pi k_x) + \cos(2\pi k_y) \Big) - \mu\,,$$

$$\Delta_{\mathbf{k}} = \Delta \Big(\cos(2\pi k_x) - \cos(2\pi k_y) \Big),$$

The ground state of the pairing Hamiltonian is the Bogoliubov vacuum

$$| \Psi_{\mathrm{DW}} \rangle = \prod_{\mathbf{k}} \Big(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} \Big) | \mathrm{vac} \rangle$$

$$= \prod_{\mathbf{k}} \exp \Big(\theta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} - \mathrm{h.c.} \Big) | \mathrm{vac} \rangle\,,$$



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Bogoliubov transformation

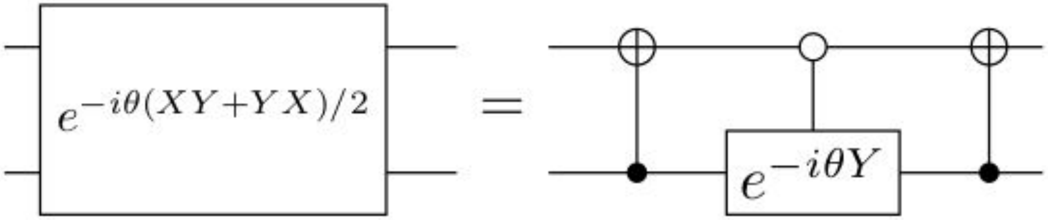
Fermionic operators can be mapped to Pauli operators using the JWT by choosing a **specific ordering**:

$$c_j^\dagger \mapsto \frac{1}{2}\left(X_j - iY_j\right) Z_1 \cdots Z_{j-1}$$

$$c_j \mapsto \frac{1}{2}\left(X_j + iY_j\right) Z_1 \cdots Z_{j-1}$$

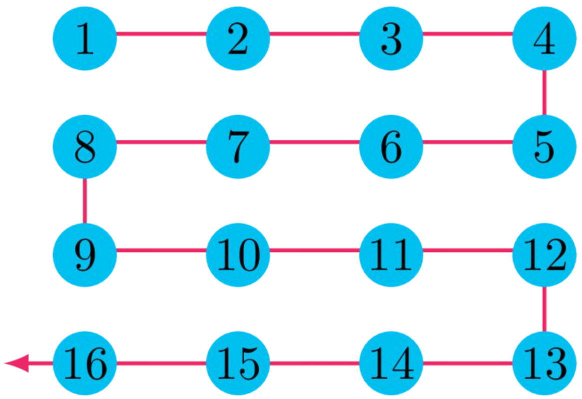
Quantum circuit to implement the bogoliubov transformation.

$$i\left(c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} - \mathrm{H.c.}\right) \mapsto \frac{1}{2}\left(X_{\mathbf{k}\uparrow} Y_{-\mathbf{k}\downarrow} + Y_{\mathbf{k}\uparrow} X_{-\mathbf{k}\downarrow}\right)$$



2D fermionic Fourier transformation (FFT)

We encode the qubits using the JWT where the sites are ordered in a row-major order.



The FFT allows for efficient state preparation and measurements. The 2D FFT can be factorized into two 1D FTs:

$$\mathcal{F} = \mathcal{F}_x \mathcal{F}_y = \mathcal{F}_x \Gamma^\dagger \mathcal{F}_y^b \Gamma \,,$$

where $\mathcal{F}_x$ is easy to implement while $\mathcal{F}_y$ is hard due to the nonlocal parity operators.

By applying Γ to $\mathcal{F}_y^b$, we avoid the parity operators, which reduces the circuit depth from $O(N)$ to $O(N^{0.5})$.

Z Jiang, KJ Sung, K Kechedzhi, VN Smelyanskiy, S Boixo, “Quantum algorithms to simulate many-body physics of correlated fermions”, Physical Review Applied 9 (4), 044036 (2018)

Gate counts

Simulate the Fermi-Hubbard model of size 6 × 6 with a qubit array of size 6 × 13:

BCS ground state: 36 × 3 = 108

Bare Givens rotations

Store parities in ancilla qubits

Parity basis

Fourier transformation: 2 × 5 × 6 × 6 + 8 × 36 + 2 × 60 + 4 × 37 = 916

CZ gates

10 Trotter steps: 10 × (72 + 72 × 5 + 36) = 4680

SWAP gates: double the total gate count of the horizontal terms

Total number of gates: ~ 7000

Sachdev-Ye-Kitaev Model

$$H = \frac{1}{4 \cdot 4!} \sum_{p,q,r,s=0}^{N-1} J_{pqrs} \gamma_p \gamma_q \gamma_r \gamma_s$$

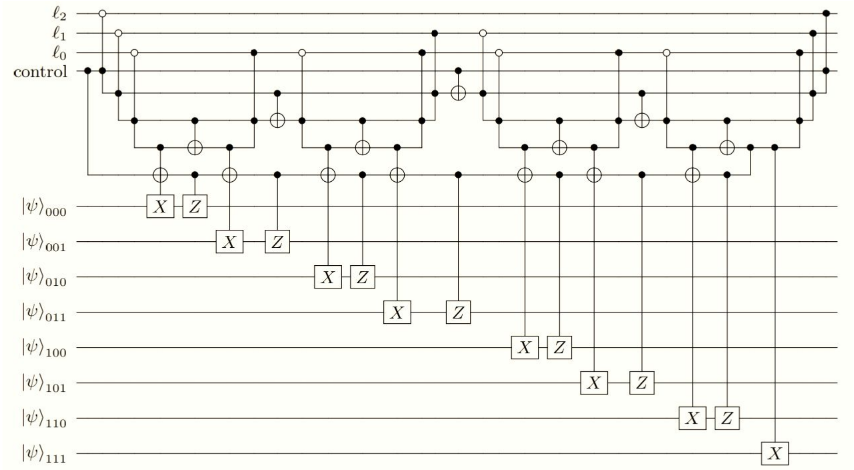
where γ_p are Majorana fermion mode operators.

$$\gamma_p = X_p \cdot Z_{p-1} \cdot Z_{p-2} \cdots Z_0$$

The complexity of the model might be simplified by using the decomposition

$$H = \lambda \sum_{\ell=0}^{L-1} \alpha_\ell \beta_\ell^* H_\ell = \sum_{\ell=0}^{L-1} w_\ell H_\ell$$

The Gaussian coefficients in the SYK model can be generated by a random quantum circuit. Each Majorana operator in *H* can be implemented in a controlled fashion.



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R. Babbush, D.Berry, and H. Neven, "Quantum Simulation of the Sachdev-Ye-Kitaev Model by Asymmetric Qubitization", arXiv:1806.02793.

Lattice gauge field theory and qubit locality

$$c_{2j} = f_j^\dagger + f_j \,, \quad c_{2j+1} = i(f_j^\dagger - f_j)$$

$$\eta_k = i c_{2k+1} c_{2k} \,, \qquad \text{for each vertex } k \in V,$$

$$\xi_{jk} = i c_{2j} c_{2k} \,, \qquad \text{for each edge } (j,\, k) \in E,$$

For each closed path we have

$$(-i)^\ell \, \xi_{k_0,k_{\ell-1}} \xi_{k_{\ell-1},k_{\ell-2}} \cdots \xi_{k_2,k_1} \xi_{k_1,k_0} = \mathbb{1}$$

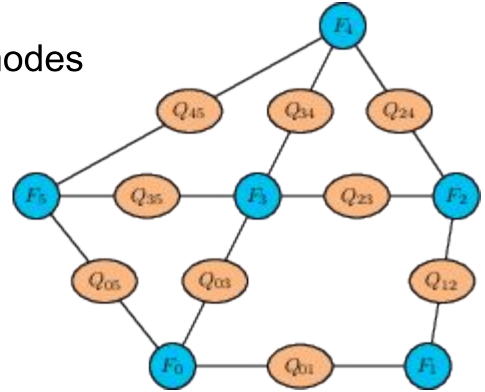
The quadratic Majorana operators can be mapped to local qubit operators.

$$\eta_k \mapsto \prod_{j:\,(j,k)\in E} Z_{jk} \,,$$

$$\xi_{jk} \mapsto \epsilon_{jk} X_{jk} \prod_{\substack{l:\,(l,k)<(j,k);\\ (l,k)\in E}} Z_{lk} \prod_{\substack{m:\,(m,j)<(k,j);\\ (m,j)\in E}} Z_{mj} \,,$$

Blue: fermionic modes

Orange: qubits



S. B. Bravyi and A. Y. Kitaev, “Fermionic Quantum Computation,” Annals of Physics 298, 210 (2002).

R. C. Ball, “Fermions without fermion fields,” Phys. Rev. Lett. 95, 176407 (2005).

F. Verstraete and J. I. Cirac, “Mapping local Hamiltonians of fermions to local Hamiltonians of spins,” Journal of Statistical Mechanics: Theory and Experiment 2005, P09012 (2005).

E. Zohar and J. Cirac, “Eliminating fermionic matter fields in lattice gauge theories”, arXiv:1805.05347 (2018)


It is better to take many small steps in the right direction than to make a great leap forward only to stumble backward.
--Louis Sachar
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